Info to SimplexNumerica

Smith Diagram Chart

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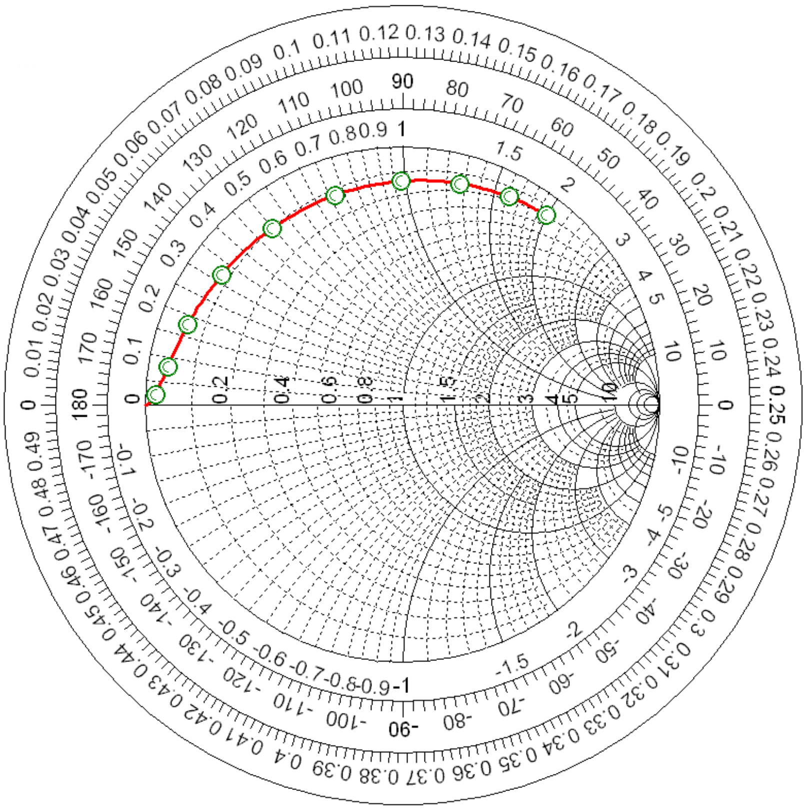
Smith Diagram

In electro-technology, particularly in the high frequency, antenna and microwave engineering, the Smith diagram is very often used, because it is an extraordinary chart for visualizing the impedance of a transmission line as a function of frequency. Smith diagrams can also be used to improve the understanding of the transmission impedance behavior and impedance matching.

Smith diagrams were originally developed around 1940 by Phillip Smith as a useful tool to make equations involving transmission lines easier to handle. See, for example, the input impedance equation for a load connected to a transmission line of length L and characteristic impedance Z0. With modern computers, the Smith chart is no longer used to simplify the calculation of equations for transmission lines; however, its value for visualizing the impedance of an antenna or transmission line has not diminished.

A Smith diagram built with SimplexNumerica is shown in the next figure



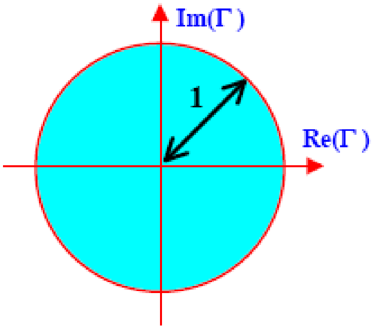
The Smith diagram is the result of the mapping of the right Gauss' number level into a circle area.

The input of the data is automatically standardized on the reference resistance (e.g. 50 ohms). Therefore, for the point (1,0) an input of x = 50 and y = 0 are necessary. The points are marked by the markers in the Smith diagram. The point (50,0) corresponds to the origin of the coordinate system in the w-level. They can represent all points of the right z-half plane in the Smith diagram.

Smith Diagram Chart

Theory

The Smith diagram is one of the most useful graphical tools for high frequency circuit applications. The chart provides a clever way to visualize complex functions and it continues to endure popularity decades after its original conception.



From a mathematical point of view, the Smith diagram is simply a representation of all possible complex impedances with respect to coordinates defined by the reflection coefficient.

The domain of definition of the reflection coefficient is a circle of radius 1 in the complex plane. This is also the domain of the Smith diagram.

The goal of the Smith diagram is to identify all possible impedances on the domain of existence of the reflection coefficient. To do so, we start from the general definition of line impedance (which is equally applicable to the load impedance)

$$Z(d) = \frac{V(d)}{I(d)} = Z_0 \frac{1+\Gamma(d)}{1-\Gamma(d)}$$

This provides the complex function

$$Z(d) = f\left\{\mathrm{Re}(\Gamma), \mathrm{Im}(\Gamma)\right\}$$

we want to graph. It is obvious that the result would be applicable only to lines with exactly characteristic impedance Z_0.

In order to obtain universal curves, we introduce the concept of normalized impedance

$$z(d) = \frac{Z(d)}{Z_0} = \frac{1+\Gamma(d)}{1-\Gamma(d)}$$

The normalized impedance is represented on the Smith diagram by using families of curves that identify the normalized resistance r (real part) and the normalized reactance x (imaginary part)

$$z(d) = \mathrm{Re}(z) + j\,\mathrm{Im}(z) = r + jx$$

Let's represent the reflection coefficient in terms of its coordinates

$$\Gamma(d) = \mathrm{Re}(\Gamma) + j\,\mathrm{Im}(\Gamma)$$

Now we can write

$$\begin{aligned} r + jx &= \frac{1+\mathrm{Re}(\Gamma)+j\,\mathrm{Im}(\Gamma)}{1-\mathrm{Re}(\Gamma)-j\,\mathrm{Im}(\Gamma)} \\ &= \frac{1-\mathrm{Re}^2(\Gamma)-\mathrm{Im}^2(\Gamma)+j2\,\mathrm{Im}(\Gamma)}{(1-\mathrm{Re}(\Gamma))^2+\mathrm{Im}^2(\Gamma)} \end{aligned}$$

Smith Diagram Chart

The real part gives

Add a quantity equal to zero

$$r = \frac{1 - \mathrm{Re}^2(\Gamma) - \mathrm{Im}^2(\Gamma)}{(1 - \mathrm{Re}(\Gamma))^2 + \mathrm{Im}^2(\Gamma)}$$

$$r(\mathrm{Re}(\Gamma) - 1)^2 + \left(\mathrm{Re}^2(\Gamma) - 1\right) + r\,\mathrm{Im}^2(\Gamma) + \mathrm{Im}^2(\Gamma) + \underbrace{\frac{1}{1+r} - \frac{1}{1+r}}_{=0} = 0$$

$$\left[r(\mathrm{Re}(\Gamma) - 1)^2 + \left(\mathrm{Re}^2(\Gamma) - 1\right) + \frac{1}{1+r}\right] + (1+r)\mathrm{Im}^2(\Gamma) = \frac{1}{1+r}$$

$$(1+r)\left[\mathrm{Re}^2(\Gamma) - 2\,\mathrm{Re}(\Gamma)\frac{r}{1+r} + \frac{r^2}{(1+r)^2}\right] + (1+r)\mathrm{Im}^2(\Gamma) = \frac{1}{1+r}$$

$$\Rightarrow \quad \left[\mathrm{Re}(\Gamma) - \frac{r}{1+r}\right]^2 + \mathrm{Im}^2(\Gamma) = \left(\frac{1}{1+r}\right)^2$$

Equation of a circle

The imaginary part gives

Multiply by x and add a quantity equal to zero

$$x = \frac{2\,\mathrm{Im}(\Gamma)}{(1 - \mathrm{Re}(\Gamma))^2 + \mathrm{Im}^2(\Gamma)}$$

$$x^2\left[(1 - \mathrm{Re}(\Gamma))^2 + \mathrm{Im}^2(\Gamma)\right] - 2x\,\mathrm{Im}(\Gamma) + \underbrace{1 - 1}_{=0} = 0$$

$$\left[(1 - \mathrm{Re}(\Gamma))^2 + \mathrm{Im}^2(\Gamma)\right] - \frac{2}{x}\mathrm{Im}(\Gamma) + \frac{1}{x^2} = \frac{1}{x^2}$$

$$(1 - \mathrm{Re}(\Gamma))^2 + \left[\mathrm{Im}^2(\Gamma) - \frac{2}{x}\mathrm{Im}(\Gamma) + \frac{1}{x^2}\right] = \frac{1}{x^2}$$

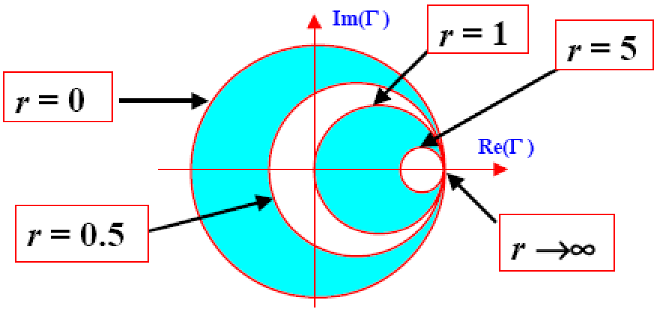
$$\Rightarrow \quad (\mathrm{Re}(\Gamma) - 1)^2 + \left[\mathrm{Im}(\Gamma) - \frac{1}{x}\right]^2 = \frac{1}{x^2}$$

Equation of a circle

The result for the real part indicates that on the complex plane with coordinates (Re(Γ), Im(Γ)) all the possible impedances with a given normalized resistance r are found on a circle with

$$\text{Center} = \left\{\frac{r}{1+r},\, 0\right\} \qquad \text{Radius} = \frac{1}{1+r}$$

As the normalized resistance r varies from 0 to ∞, we obtain a family of circles completely contained inside the domain of the reflection coefficient | Γ | = 1.

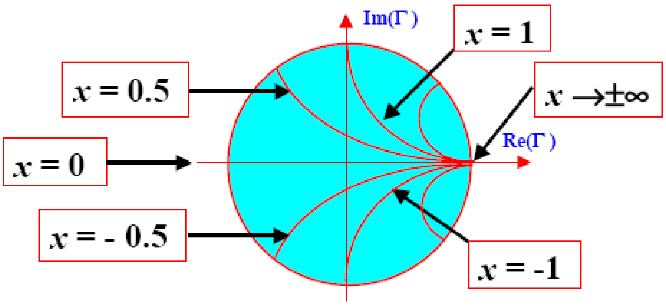


Smith Diagram Chart

The result for the imaginary part indicates that on the complex plane with coordinates (Re(Γ), Im(Γ)) all the possible impedances with a given normalized reactance x are found on a circle with

$$\text{Center} = \left\{1\,,\ \frac{1}{x}\right\} \qquad\qquad \text{Radius} = \frac{1}{x}$$

As the normalized reactance x varies from -8 to 8, we obtain a family of arcs contained inside the domain of the reflection coefficient | Γ | = 1 .



The Smith diagram can be used for line admittances, by shifting the space reference to the admittance location. After that, one can move on the chart just reading the numerical values as representing admittances. Let's review the impedance-admittance terminology:

Impedance = Resistance + j Reactance

$$Z \;=\; R \;+\; jX$$

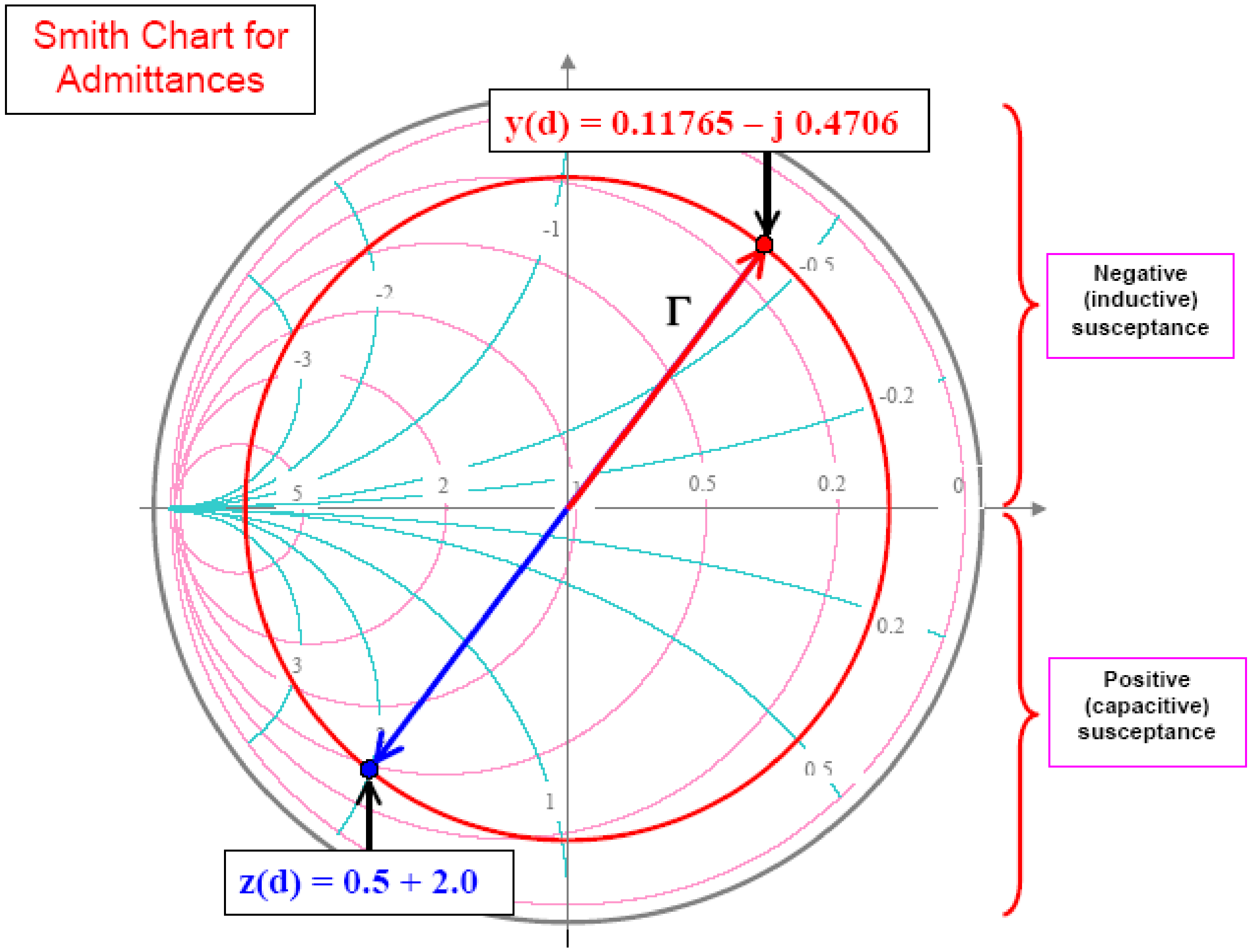
Impedance, denoted by Z, is an expression of the opposition that an electronic component, circuit, or system offers to AC (alternating current). Impedance is comprised of two independent scalar (one-dimensional) phenomena: resistance and reactance. Both of these quantities are expressed in ohms.

Admittance = Conductance + j Susceptance

$$Y \;=\; G \;+\; jB$$

On the impedance chart, the correct reflection coefficient is always represented by the vector corresponding to the normalized impedance. Charts specifically prepared for admittances are modified to give the correct reflection coefficient in correspondence of admittance.

Smith Diagram Chart



$$z = r + jx \qquad y = g + jb = \frac{1}{r + jx}$$

$$y = \frac{r - jx}{(r + jx)(r - jx)} = \frac{r - jx}{r^2 + x^2}$$

$$\Rightarrow \qquad g = \frac{r}{r^2 + x^2} \qquad b = -\frac{x}{r^2 + x^2}$$

Since related impedance and admittance are on opposite sides of the same Smith diagram, the imaginary parts always have different sign. Therefore, a positive (inductive) reactance corresponds to a negative (inductive) susceptance, while a negative (capacitive) reactance corresponds to a positive (capacitive) susceptance. Numerically, we have the left relationship.

Smith Diagram in SimplexNumerica

SimplexNumerica has build-in templates for Smith Diagrams. They are simple to use by Drag & Drop from the Thumbnail window. Every single component of the Smith Diagram can be changed in the property window. As an example, here the disassembling of a Smith Diagram.